The Coulomb sum and proton-proton correlations in few-body nuclei

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Abstract

For simple models of the nuclear charge operator, measurements of the Coulomb sum and the charge form factor of a nucleus directly determine the proton-proton correlations. We examine experimental results obtained for few-body nuclei at Bates and Saclay using models of the charge operator that include both one- and two-body terms. Previous analyses using one-body terms only have failed to reproduce the experimental results. However, we find that the same operators which have been used to successfully describe the charge form factors also produce substantial agreement with measurements of the Coulomb sum.

Typeset using REVTEX

It has long been known that the integrated strength of the longitudinal response function measured in inclusive electron scattering (the Coulomb sum rule) is related to the Fourier transform of the proton-proton distribution function (PPDF) in the nuclear ground state [1]. A crucial assumption in obtaining this relation is that the nuclear charge distribution arises solely from the protons. As the PPDF is sensitive to the short-range proton-proton correlations, its experimental determination can provide direct information on both the strength of the correlations in the nuclear medium and the size of the repulsive core in the nucleon-nucleon interaction.

Beck [2] has recently obtained an experimental PPDF from the Bates [3] and Saclay [4] longitudinal data on ^{3}He. His analysis has shown that a large discrepancy exists between the experimental PPDF and that calculated [5] from an essentially exact Faddeev wave function [6] corresponding to a realistic Hamiltonian with the Argonne v_{14} [7] two-nucleon and Urbana VII [8] three-nucleon interaction models. Specifically, he found that the experimental PPDF has a zero at lower momentum transfer and a far greater magnitude in the region of the second maximum than the calculated PPDF. It is important to point out that Faddeev calculations based on different realistic two-nucleon interactions all give very similar results, as reported by Doyle *et al.* [9]. Beck's analysis implies that the experimental PPDF is smaller at short distances than the calculated PPDF, thus suggesting that the proton-proton interaction has a stronger repulsion than present models would indicate.

In this letter we analyze the longitudinal response data on ^{3}H, ^{3}He, and ^{4}He obtained at Bates [3,10] and on ^{3}He and ^{4}He obtained at Saclay [4,11]. The Coulomb sum is defined as

$$S_L(k) = \frac{1}{\mathrm{Z}} \int_{\omega_{el}^+}^{\infty} d\omega \; \frac{R_L(k,\omega)}{[G_{E,p}(k,\omega)]^2} \quad , \tag{1}$$

where k and ω are the momentum and energy transfers, $R_L(k,\omega)$ is the longitudinal response, $G_{E,p}$ is the proton electric form factor (the Höhler parameterization [12] is used in the present work), and ω_{el} is the energy of the recoiling A-nucleon system with Z protons. It can be expressed as

$$S_L(k) = \frac{1}{\mathrm{Z}} < 0|\rho_L^\dagger(\mathbf{k})\rho_L(\mathbf{k})|0> - \frac{1}{\mathrm{Z}}| < 0|\rho_L(\mathbf{k})|0> |^2$$

$$\equiv 1 + \rho_{LL}(k) - \mathrm{Z}\frac{|F_L(k)|^2}{[G_{E,p}(k,\omega_{el})]^2} \quad , \tag{2}$$

where $|0>$ is the ground state of the nucleus, $F_L(k)$ is the charge form factor normalized as $F_L(k=0)=1$, and a longitudinal-longitudinal distribution function (LLDF) has been defined as

$$\rho_{LL}(k) \equiv \frac{1}{\mathrm{Z}}\int \frac{d\Omega_k}{4\pi} < 0 \mid \rho_L^\dagger(\mathbf{k})\rho_L(\mathbf{k}) \mid 0 > -1 \quad . \tag{3}$$

In this work we assume that the nuclear charge operator $\rho_L(\mathbf{k})$ consists of one- and two-body parts

$$\rho_L(\mathbf{k}) = \rho_{L,1}(\mathbf{k}) + \rho_{L,2}(\mathbf{k}) \quad . \tag{4}$$

The one-body part includes, in addition to the dominant proton contribution, the neutron contribution and the Darwin-Foldy and spin-orbit relativistic corrections to the single-nucleon charge operator

$$\rho_{L,1}(\mathbf{k}) = \sum_{i=1,\mathrm{A}} e^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}_i}\left[X_i - \mathrm{i}\frac{1}{4m^2}Y_i\,\mathbf{k}\cdot(\sigma_i\times\mathbf{p}_i)\right], \tag{5}$$

$$X_i = \frac{1}{(1+\bar{k}^2/4m^2)^{\frac{1}{2}}}\left[\frac{1}{2}\,(1+\tau_{z,i}) \;+\; \frac{G_{E,n}(\bar{k}^2)}{G_{E,p}(\bar{k}^2)}\,\frac{1}{2}\,(1-\tau_{z,i})\right], \tag{6}$$

$$Y_i = \frac{2}{(1+\bar{k}^2/4m^2)^{\frac{1}{2}}}\left[\frac{G_{M,p}(\bar{k}^2)}{G_{E,p}(\bar{k}^2)}\,\frac{1}{2}\,(1+\tau_{z,i}) + \frac{G_{M,n}(\bar{k}^2)}{G_{E,p}(\bar{k}^2)}\,\frac{1}{2}\,(1-\tau_{z,i})\right] - X_i\;, \tag{7}$$

where $\bar{k}^2 \equiv k^2 - (k^2/2m)^2$ is the four-momentum transfer corresponding to the quasielastic peak, and $G_{E,n}$, $G_{M,n}$ and $G_{M,p}$ are the neutron electric, neutron magnetic and proton magnetic form factors, respectively, evaluated at $\bar{k}^2$. The Darwin-Foldy correction is taken into account by the factor $1/(1+\bar{k}^2/4m^2)^{\frac{1}{2}}$ as suggested by Friar [13]. Note that because of the definition of S_L in Eq. (1), the charge operator in Eq. (4) is divided by $G_{E,p}$.

The two-body part contains contributions associated with pion, ρ- and ω-meson exchanges, and the $\rho\pi\gamma$ and $\omega\pi\gamma$ mechanisms [14]. In the momentum transfer range of interest ($k \lesssim 600$MeV/c) the pion term is by far the most important. For example, the contributions

to the A = 3 and 4 charge form factors of the vector meson terms are at least one order of magnitude smaller. The pion term is given by

$$\rho_{L,\pi}(\mathbf{k}) = \frac{3\mathrm{i}}{2m} \sum_{i<j=1,\mathrm{A}} I_\pi(r_{ij}) \left[Z_j \, \sigma_i \cdot \mathbf{k} \;\; \sigma_j \cdot \hat{\mathbf{r}}_{ij} \, e^{\mathbf{i}\mathbf{k}\cdot\mathbf{r}_i} + i \rightleftarrows j \right] \quad , \tag{8}$$

$$Z_j \equiv \frac{F_1^s(\bar{k}^2)}{G_{E,p}(\bar{k}^2)} \tau_i \cdot \tau_j + \frac{F_1^v(\bar{k}^2)}{G_{E,p}(\bar{k}^2)} \tau_{z,j} \quad , \tag{9}$$

$$I_\pi(r) = -\frac{1}{3m_\pi^2 r^2} \left(\frac{f_\pi^2}{4\pi} \right) \left\{ (1 + m_\pi r) e^{-m_\pi r} - (1 + \Lambda_\pi r) e^{-\Lambda_\pi r} - \frac{1}{2} \left[1 - \left(\frac{m_\pi}{\Lambda_\pi} \right)^2 \right] (\Lambda_\pi r)^2 e^{-\Lambda_\pi r} \right\} , \tag{10}$$

where m_π and f_π are the pion mass and the πNN coupling constant, respectively, with $f_\pi^2/4\pi = .081$. The form factor at the πNN vertex Λ_π is chosen to be large ($\Lambda_\pi = 2$ GeV), as suggested by an analysis of the pseudoscalar component of the Argonne v_{14} interaction [15]. F_1^s and F_1^v are the Dirac isoscalar and isovector nucleon form factors. The charge operator given in Eq. (4) gives an excellent description of the charge form factors of ^{3}H, ^{3}He, and ^{4}He in calculations based on essentially exact Faddeev (A = 3) [16] and Green's function Monte Carlo (GFMC) (A = 4) [17] wave functions obtained from the Hamiltonian containing the Argonne v_{14} and Urbana VIII interactions. (This Hamiltonian correctly reproduces the experimental binding energies of A = 3 and 4 nuclei in Faddeev and GFMC calculations.)

In order to experimentally determine the LLDF in Eq. (2) it is necessary to measure both the charge form factor F_L and the Coulomb sum S_L. For the charge form factors of ^{3}H, ^{3}He, and ^{4}He we have used accurate fits to the world data provided to us by Sick [18]. As the longitudinal response can be measured only up to some $\omega_{\max} < k$ by inclusive electron scattering, it is necessary to estimate the contribution of the unobserved strength for $\omega > \omega_{\max}$ in order to obtain the Coulomb sum. We have assumed that for $\omega > \omega_{\max}$ the longitudinal response can be parameterized as

$$R_L(k, \omega > \omega_{\max}) = R_L(k, \omega_{\max}; \exp) \left(\frac{\omega_{\max}}{\omega} \right)^{\alpha(k)} \quad . \tag{11}$$

This form has been suggested by a study of the high ω-behavior of the deuteron longitudinal response, which can be accurately calculated [19]. It has been found that for the Argonne v_{14} interaction, the power $\alpha(k)$ in the deuteron is in the range 3.0–3.5 for k between 200–600 MeV/c. In the A = 3 and 4 nuclei it is determined by requiring that the energy-weighted sum rule $W_L(k)$,

$$W_L(k) = \frac{1}{\mathrm{Z}} \int_{\omega_{el}^+}^{\infty} d\omega \ \omega \ \frac{R_L(k,\omega)}{[G_{E,p}(k,\omega)]^2} \quad , \tag{12}$$

reproduces that calculated as

$$W_L(k) = \frac{1}{\mathrm{Z}} < 0|\rho_L^\dagger(\mathbf{k})[H,\rho_L(\mathbf{k})]|0> - \frac{1}{\mathrm{Z}}\omega_{el}| < 0|\rho_L(\mathbf{k})|0> |^2, \tag{13}$$

by exact Monte Carlo methods. Here H is the Hamiltonian with the Argonne v_{14} and Urbana VIII interactions, and ρ_L is the operator given in Eq. (4). The parameter α is typically found to be in the range 2.8–3.5 (2.6–3.1) for the A = 3 (A = 4) nuclei and k = 200–600 MeV/c, and does not depend significantly on the value $\omega_{\rm max}$ chosen. The present analysis differs from that reported in Refs. [20,21] in two respects. First, in Ref. [20] the tail contribution to $S_L(k)$ is estimated by parameterizing the high-ω tail of the response as a sum of two decreasing exponentials required to join the data smoothly and to satisfy the calculated energy- and energy-square-weighted sum rules. It should be noted that the values for α reported above suggest that the energy-square-weighted sum rule may not exist. Secondly, the energy-weighted sum rule has been calculated here with a charge operator that includes both one- and two-body components rather than the proton contribution only as in Refs. [20,21]. The two-body components (predominantly that associated with pion exchange) lead to an enhancement of $W_L(k)$ of 10% (6.0%), 8.6% (4.3%), 7.5% (3.4%) in ^{3}H (^{3}He) and 9.1%, 7.8% and 7.4% in ^{4}He at k = 300, 400, 500 MeV/c, respectively. The dominant kinetic energy contribution, which is exactly given by $k^2/2m$ when only protons are included in ρ_L, is little affected by the relativistic corrections and two-body terms. However, the latter enhance the leading interaction contributions associated with isospin-exchange spin and tensor components.

As a consequence of these differences, the present analysis yields values for $S_L(k)$ that are slightly larger ($\lesssim$2%) than those published in Ref. [20] for ^{3}H and ^{3}He, the only data for which a comparison is possible. The final analysis of the Bates data on ^{4}He, published in Ref. [10], has given a separated longitudinal response that is somewhat smaller in the quasielastic peak than that used in Ref. [20] to obtain $S_L(k)$.

The experimental LLDF obtained for ^{3}H, ^{3}He, and ^{4}He are compared with theory in Figs. 1–3. The errors in the experimental LLDF are dominated by those in the Coulomb sum. The latter has two sources: the first, from the measured portion of $S_L(k)$, denoted as $S_L(k;\exp)$; the second, from the tail contribution, denoted as $S_L(k;\text{tail})$. The error on $S_L(k;\exp)$ has been estimated by adding in quadrature the random errors on the measured longitudinal response function and by further assuming the systematic error to be as large as the random error so obtained. The error on $S_L(k;\text{tail})$ has been estimated by assuming it to be given by $S_L(k;\text{tail})\times\ \Delta R_L(k,\omega_{\max})/R_L(k,\omega_{\max})$, where ΔR_L is the experimental error on $R_L(k,\omega)$ at $\omega=\omega_{\max}$ (typically ∼20-30% of $R_L(k,\omega_{\max})$).

The theoretical curves in Figs. 1-3 have been obtained by exact Monte Carlo evaluation of the expectation value in Eq. (3). We have used exact Faddeev (A=3) and GFMC (A=4) wave functions, again corresponding to the Argonne v_{14} plus Urbana VIII interaction models. The LLDF obtained from the Bates and Saclay data on ^{3}He and ^{4}He are consistent with each other, within errors, and are in good agreement with the results of calculations in which both one- and two-body terms are included in the charge operator. In particular, the position of the zero and magnitude of the second maximum are well reproduced by these calculations. The results obtained by neglecting the contributions due to the two-body terms or by keeping only the proton contributions in ρ_L are in poor agreement with the data: the zero is shifted to higher k's and the strength in the second maximum is greatly underestimated, as found by Beck [2]. We also note that in the calculation of ^{3}H, ^{3}He and ^{4}He charge form factors, inclusion of the two-body components in ρ_L is crucial for correctly reproducing the experimental data in the diffraction minimum region. The minima are located at $k \simeq 630$ MeV/c in the ^{3}He and ^{4}He charge form factors. However,

they occur at significantly lower momentum, $k \simeq 370$ MeV/c, in the ^{3}He and ^{4}He LLDF, thus enhancing the importance of the relativistic and meson-exchange corrections at low momentum transfers.

In ^{3}H the LLDF calculated in the approximation in which only protons are considered vanishes identically. However, the experimental LLDF extracted from the Bates data is different from that obtained in the full calculation. This discrepancy is also found in the Coulomb sum rule: the experimental $S_L(k)$ (including the tail contribution) is larger by $\sim$ 10% in the $k = 350$–500 MeV/c range than the theoretical one. However, the ^{3}H experimental charge form factor is well reproduced by the present theory.

To summarize, the LLDF has been calculated in the A = 3 and 4 nuclei with exact Faddeev and GFMC wave functions obtained from a realistic Hamiltonian containing the Argonne v_{14} two-nucleon and Urbana VIII three-nucleon interaction models. The charge operator has been taken to include, in addition to the dominant proton contribution, also the neutron contribution, the Darwin-Foldy and spin-orbit relativistic corrections, and two-body terms associated with meson exchanges. Within this framework good agreement has been obtained between the calculated and experimental ^{3}He and ^{4}He LLDF. However, large discrepancies remain between the calculated and experimental ^{3}H LLDF.

The present ^{3}He and ^{4}He results indicate that, because of the complicated nature of the coupling between a longitudinal virtual photon and the nucleus (even at low momentum transfers), the LLDF extracted from inclusive electron scattering data cannot provide direct information on the strength of the proton-proton repulsive interaction at short range. Therefore, the experimental evidence based on inclusive longitudinal data for proton-proton short-range correlations remains elusive.

ACKNOWLEDGMENTS

We wish to thank A. Bernstein, J. Morgenstern, S. Platchkov, and I. Sick for their kind cooperation in providing us with tables of the elastic and inelastic data on the A = 3 and 4

nuclei. The work of R. S. is supported by the Istituto Nazionale di Fisica Nucleare, Italy, that of R. B. W. by the U. S. Department of Energy, Nuclear Physics Divsion, under Contract No. W-31-109-ENG-38, and that of J. C. by the U. S. Department of Energy.

REFERENCES

[1] K. W. McVoy and L. Van Hove, Phys. Rev. **125**, 1034 (1962).

[2] D. H. Beck, Phys. Rev. Lett. **64**, 268 (1990).

[3] K. Dow *et al.*, Phys. Rev. Lett. **61**, 1706 (1988).

[4] C. Marchand *et al.*, Phys. Lett. **153B**, 29 (1985).

[5] R. Schiavilla *et al.*, Nucl. Phys. **A473**, 267 (1987).

[6] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. C **33**, 1740 (1986).

[7] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).

[8] R. Schiavilla, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A449**, 219 (1986).

[9] B. Doyle, B. Goulard, and G. Cory, Phys. Rev. C **45**, 1444 (1992).

[10] K. F. von Reden *et al.*, Phys. Rev. C **41**, 1084 (1990).

[11] J. F. Danel *et al.*, unpublished.

[12] G. Höhler *et al.*, Nucl. Phys. **B114**, 505 (1976).

[13] J. L. Friar, Ann. Phys. (NY) **81**, 332 (1973).

[14] R. Schiavilla, V. R. Pandharipande, and D. O. Riska, Phys. Rev. C **41**, 309 (1990).

[15] R. Schiavilla, V. R. Pandharipande, and D. O. Riska, Phys. Rev. C **40**, 2294 (1989).

[16] R. B. Wiringa, Phys. Rev. C **41**, 1585 (1991).

[17] J. Carlson, Phys. Rev. C **38**, 1879 (1988); Nucl. Phys. **A508**, 141c (1990).

[18] I. Sick, private communication.

[19] J. Carlson and R. Schiavilla, Phys. Rev. Lett. **68**, 3682 (1992).

[20] R. Schiavilla, V. R. Pandharipande, and A. Fabrocini, Phys. Rev. C **40**, 1484 (1989).

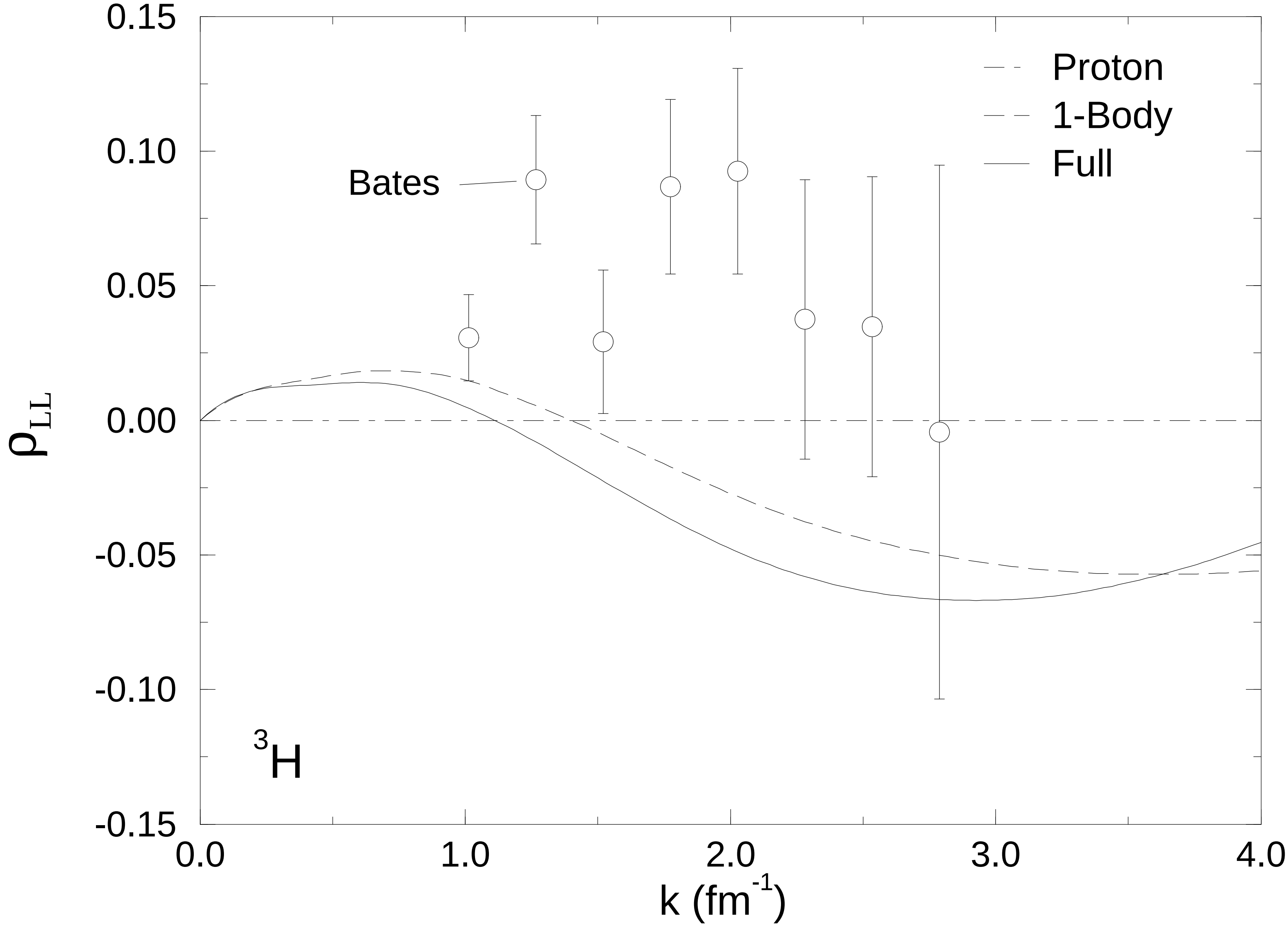
[21] R. Schiavilla, A. Fabrocini, and V. R. Pandharipande, Nucl. Phys. **A473**, 290 (1987).

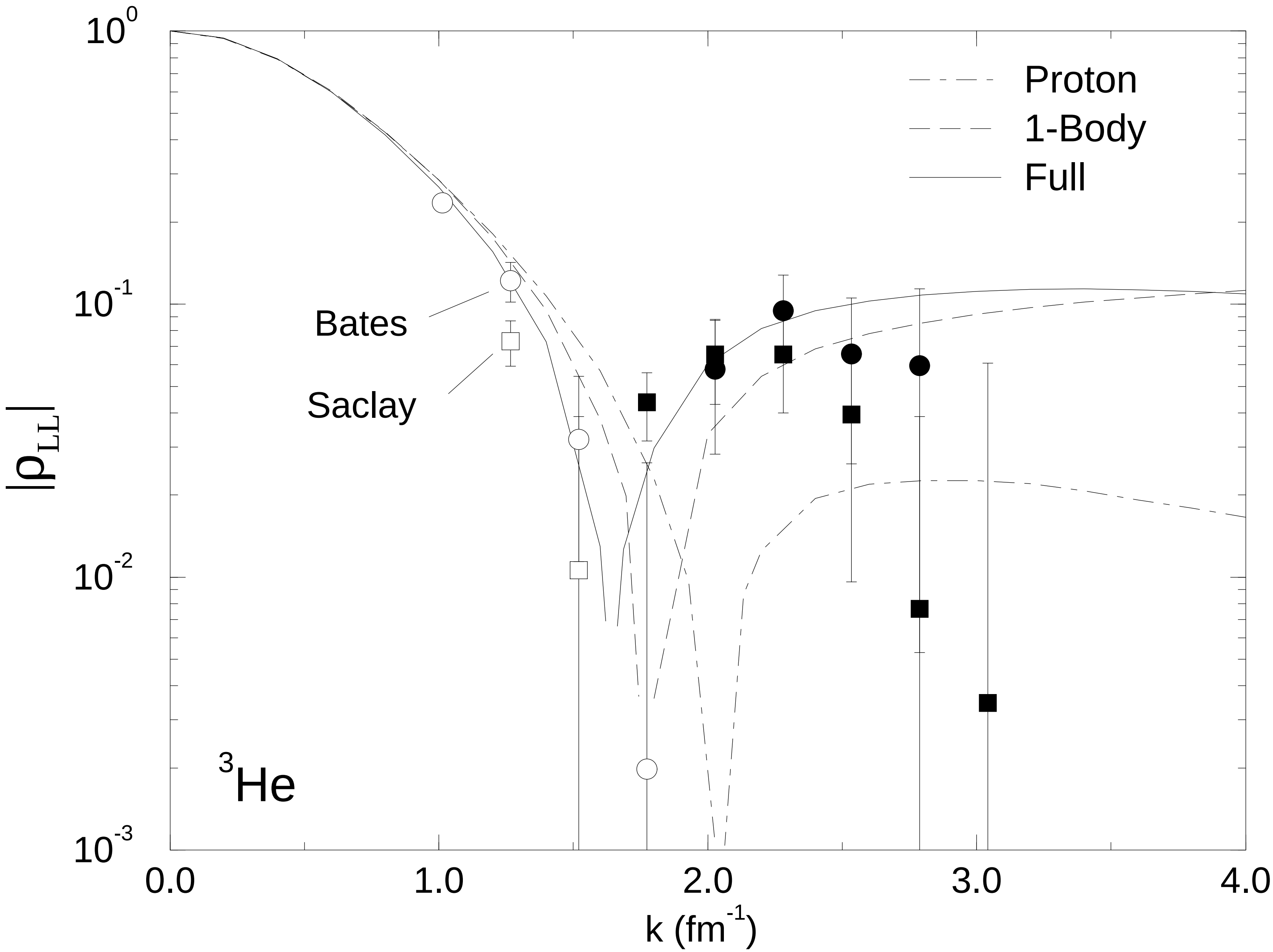
FIGURES

FIG. 1. Experimental and theoretical longitudinal-longitudinal distribution functions in ^{3}He. Circles (squares) denote Bates (Saclay) data; solid symbols denote negative values. The curves labelled proton, 1-body, and full show theoretical results obtained from the Faddeev wave function by including in ρ_L the proton, one-body, and one- plus two-body contributions, respectively.

FIG. 2. Same as in Fig.1 but for ^{4}He with theoretical results from the GFMC wave function.

FIG. 3. Same as in Fig.1 but for ^{3}H.


0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
ρ_{LL}
0.0
1.0
2.0
3.0
4.0
k (fm^{-1})
Proton
1-Body
Full
Bates
^{3}H



Proton
1-Body
Full
Bates
Saclay
^{3}He
$|\rho_{LL}|$
10^0
10^{-1}
10^{-2}
10^{-3}
0.0
1.0
2.0
3.0
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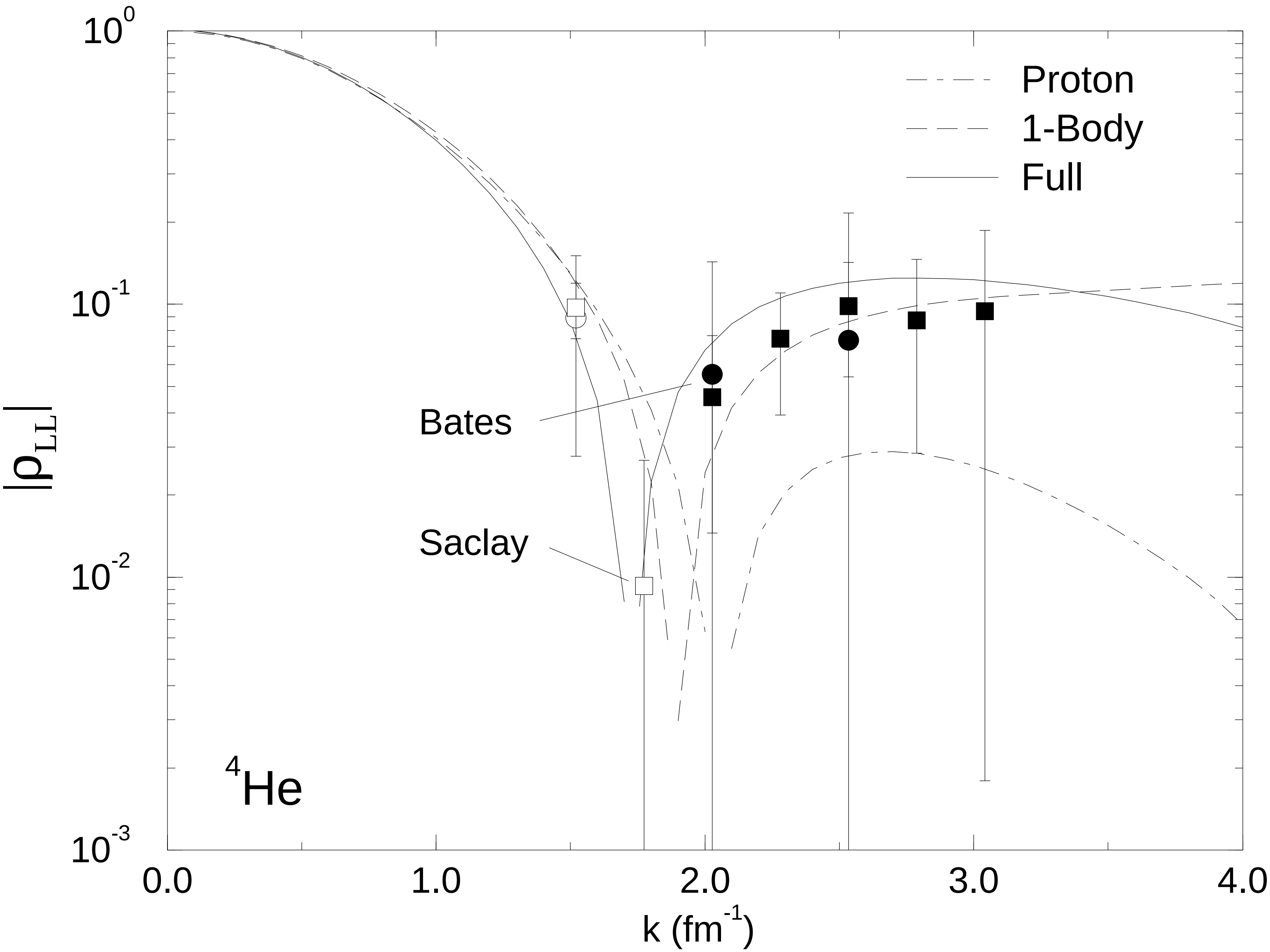
k (fm^{-1})



Proton
1-Body
Full
Bates
Saclay
^{4}He
$|\rho_{LL}|$
k (fm^{-1})
10^0
10^{-1}
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